**ISTANBUL BILGİ UNIVERSITY**

**FACULTY OF ENGINEERING AND NATURAL SCIENCES Department of Electrical-Electronics Engineering**

**EEEN 460**

**OPTIMAL CONTROL**

**FINAL EXAM**

|  |
| --- |
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| ID Number: 115200084 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
| Score |  |  |  |  |  |  |  |  |  |
| Maximum score | 20 | 10 | 10 | 10 | 10 | 10 | 10 | 20 | 100 |

**HONOR CODE**

Dear Students, we invite you to be dedicated to protect the

Integrity of this exam, as well as yours and your classmates’s work and efforts.

As a part of this dedication we ask you to read and follow the following rules.

Please

* Submit your own original work
* Avoid sharing answers with others
* Report suspected violations

Thank you for your cooperation.

**NOTE:**

**There are 8 questions in this exam. Please Note that the first and last questions are worth 20 points and the rest are 10 points. Give your answers by using this document, i.e. enter your answers to  the provided blank spaces, you may add additional pages. When you complete this exam submit it to the inbox as a Word document. Inbox will be open till 18:59.**

1. Write the performance measure index for each of the following optimal control scenarios **(20 points):**
2. A Time Optimal Controller

Suppose the objective is to reach as soon as possible to the final point in a given optimal time; then the performance measure J is given by,

J = -

In all that follows it will be assumed that the performance of a system of a system is evaluated by a measure of the form,

J = h (x ( +

h(( + ≤ h(x( +

1. A Regulator

If the regulator tries to keep the level of air in container at constant level. That’s why our formula should be, [G represents Pressure]

] dt ≤ G

1. A Target Tracker

To transfer a system from arbitrary initial state to a specified target set S in minimum time. The performance measure to be minimized is,

J = -

with the first instant of time when intersect. It is the typical example for the interception of tracking target.

J = h (x( +

It can be evaluated with this formula.

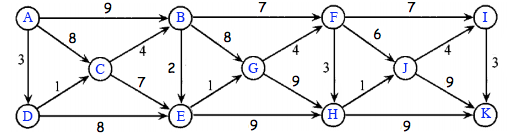
1. A Minimum Energy Controller

In this statement, we can use this equation below,

h(( + ≤ h(x( + that formula. Because a minimum energy controller trying to stay at minimum level **as closely as possible**. That is, it tries to stay the lowest point. So, the formula will be;

h(( + could be equal to 0.

1. A driver can travel on the streets shown on this map where costs of sub-paths are given. Find the optimum path from **start (A)** to **target (K)** by using dynamical programming and back-propagation, (trial & error solutions will not be accepted) **(10 points).**

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North

**Answer:**

East

West

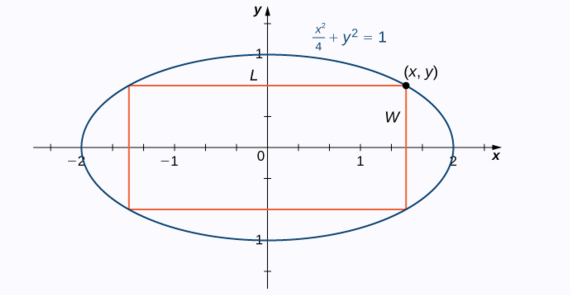
South

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Current Intersection**  **α** | **Heading** | **Next Intersection** | **Minimum cost from α to h via**  = | **Minimum cost to reach h from α** | **Optimal Heading at α** |
| **I** | **South** | **K** | **3 + 0 = 3** | **3** | **South** |
| **J** | **North-East**  **South-East** | **I**  **K** | **4 + 3 = 7**  **9 + 0 = 9** | **7** | **North-East** |
| **H** | **North-East**  **East** | **J**  **K** | **1 + 7 = 8**  **9 + 0 = 9** | **8** | **North-East** |
| **F** | **East**  **South-East**  **South** | **I**  **J**  **H** | **7 + 3 = 10**  **6 + 7 = 13**  **3 + 8 = 11** | **10** | **East** |
| **G** | **North-East**  **South-East** | **F**  **H** | **4 + 10 = 14**  **9 + 8 = 17** | **14** | **North-East** |
| **B** | **East**  **South-East**  **South** | **F**  **G**  **E** | **7 + 10 = 17**  **8 + 14 = 22**  **2 + 15 = 17** | **17** | **East**  **South** |
| **E** | **North-East**  **East** | **G**  **H** | **1 + 14= 15**  **9 + 8 = 17** | **15** | **North-East** |
| **C** | **North-East**  **South-East** | **B**  **E** | **4 + 17 = 21**  **7 + 15 = 22** | **21** | **North-East** |
| **D** | **North-East**  **East** | **C**  **E** | **1 + 21 = 22**  **8 + 15 = 23** | **22** | **North-East** |
| **A** | **East**  **South-East**  **South** | **B**  **C**  **D** | **9 + 17 = 26**  **8 + 21 = 29**  **3 + 22 = 25** | **25** | **South** |

**Thus, we have two paths with similar length because of B goes to F and E with same length. As you can see below, I have shown them with their path as first and second.**

First Path: A 🡪 D 🡪 C🡪 B 🡪 F 🡪 I 🡪 K && Second Path: A 🡪 D 🡪 C 🡪 B 🡪 E 🡪 G 🡪 F 🡪 I 🡪 K

3. In the figure below you see an ellipse which is enclosing a rectangle



y

x

The equation of ellipse is given by

Find the length(L) and width (W) of the rectangle which will maximize its area, (A). What is max(A)?

L = 2x , W = 2y (As you can see above in the figure)

Equation of Ellipse: = 1

🡪 **First Equation**

Area of rectangle: A = (2x) (2y)

A = 4xy 🡪 Squaring on both sides A2 = 16 x2y2

\*If A is maximum, then A2 is also maximum

f (x, y) = A2 = 16 x2y2

= 16 (4 – 4y2) (y2)

f(y) = 16 (4y2 – 4y4) 🡪 **Second Equation**

condition for maximum y is as you can see 🡪 f’(y) = 0

f(y) = 16 (4y2 – 4y4)

on differentiating the equation

f’(y) = 16 (8y – 16y3) = 0

8y – 16y3 = 0

8y (1-2y2) = 0 🡪 so y = 0 and y = 1/

\*If y = 0 then, area won’t be maximum

For maximum area y = 1/

Thus,

Equation 1 🡪 x2 = 4 – 4y2

= 4 – 4 (1/)2

= 4 – 2

X2 = 2 🡪 x = (maximum area)

For maximum area; answer will be as you can see below:

L = 2x W = 2y

L = 2 W =

1. In the system given below (**10 points):**

Objective function is given by

**p(x,y)=250x+75y**

Maximize it by using the **graphical method**,

under the constraints:

**5x+y≤100**

**x+y≤60**

**0≤x**

**0≤y​**

1. What is **(x\*, y\*) ?**
2. What is **p\*( x\*, y\*) ?**

**Answer:**

x + y = 60 🡪 x = 0 (0,60)

🡪 y = 0 (60,0)

5x + y = 100 🡪 x = 0 (0,100)

🡪 y = 0 (20,0)

5x + y = 100 🡪 First Eq. (1)

x + y = 60 🡪 Second Eq. (2)

1. – (2) = 4x = 40 🡪 x = 10

Y = 50

(10,50) IP (Intersection Point)

**100**

(x, y) P(x, y)

(0,0) 0

(0,60) 4500

(10,50) 6250 (MAX)

(20,0) 5000

**60**



**60**

**20**

x + y ≤ 60

5x+y ≤ 100

a) X\*=10, Y\*=50

b) P\*(x\*, y\*) = 6250

1. By using **Simplex method** maximize **(10 points):**

**P = 6x + 5y**

Subject to:

**x + y ≤ 5  
3x + 2y ≤ 12**

**x≥0**

**y​≥0**

**Answer:**

P - 6x - 5y = 0

x + y + b = 5

3x + 2y + e = 12

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | P | x | y | b | e |  |
| P | 1 | -6 | -5 | 0 | 0 | 0 |
| b | 0 | 2 | 3 | 1 | 0 | 5 |
| e | 0 | 3 | 0 | 0 | 1 | 12 |

MR Test (Minimum Ratio):

5 / 1 = 5 & 12 / 4 = 3

R1 🡪 R1 + (2 \* R3)

R2 🡪 R2 - ((R2) / 3)

R3 🡪 (R3) / 3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | P | x | y | b | e |  |
| P | 1 | 0 | -1 | 0 | 2 | 24 |
| b | 0 | 0 | 1/3 | 1 | -1/3 | 1 |
| e | 0 | 1 | 2/3 | 0 | 1/3 | 4 |

And also;

R1 🡪 R1 \* 1

R2 🡪 R2 \* 3

R3 🡪 (2/3) \* R2 \* R3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | P | x | y | b | e |  |
| P | 1 | 0 | -1 | 3 | 1 | 27 |
| b | 0 | 0 | 1 | 3 | -1 | 3 |
| e | 0 | 1 | 0 | -2 | 1 | 2 |

P (x\*, y\*) = 27

Q6. A manufacturer has two machines A and B in his factory. Machines A and B are capable of being operated for at most 12 hours each. He produces only two items M and N each requiring the use of both machines. The number of hours required for producing 1 unit of each of M and N on two machines are given in the following table.

|  |  |  |
| --- | --- | --- |
|  | Number of hours required on each machine | |
| Item | A | B |
| M | 1 | 3 |
| N | 3 | 1 |

The manufacturer makes a profit of § 18 and § 12 (where § is a virtual money unit) on items M and N respectively. How many of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produced? What will be the maximum profit? **(10 points)**

3t + e = 12 🡪 t = 0 (0,12)

🡪 e = 0 (4,0)

t + 3e = 12 🡪 t = 0 (0,4)

🡪 e = 0 (12,0)

t + 3e = 12 🡪 (1)

3t + e = 12 🡪 (2)

-3 \* (2) + (1) = -8t = -24 🡪 t = 3

e = 3

(3,3) IP (Intersection P.)

**12**

**4**

P = 18 \* t + 12 \* e

t e P(t\*,e\*)

0 4 48

4 0 72

3 3 90 (MAX)

0 0 0



**12**

**4**

t + 3e = 12

3t + e = 12

**Answer:**

P(t\*,e\*) = 90

t\* = 3

e\* = 3

Q7. **(10 points)**

Using the Euler –Lagrange equation

a) Find x(t) (as an equation) which minimizes the following objective function

b) Determine the parameters in a)

if x(0)=1 and x(2)=5

🡪 Euler-Lagrange Equation

g

We need to find Eular-lagrange Equation;

g , g , g

Euler-Lagrange Equation:

If x (0) = 1; 🡪 = 1

If x (2) = 5; 🡪 =

Minimized Equation:

Q8 The first order discrete system **(20 points)**

x(k+1)=2x(k)-3u(k)

is to be transferred from initial state x(0)=4 to final state x(2)

in two states while the performance index

is minimized.

Find the optimal control sequence

x (2) =2 x (1)-3u (1)

=

= 0

= 4u(1) + 2 [2x(1)-3u(1)-10].(-3) = 0

= 4u(1) + 6 [2x(1)-3u(1)-10] = 0

22u(1) - 12x(1) + 60 = 0

u (1) = = 0

x(1) = 2x(0) - 3u(0) 🡪 (If k = 0)